

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

7 OCTOBER 2016  
9.00 A.M. – 11.00 A.M.  
(2 Hours)

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**INSTRUCTIONS TO STUDENTS**

1. This Question paper consists of **5 pages** only including the cover page with **5 Questions** and an **Appendix**.
2. There are **2 Sections**:  
**Section A** (3 Questions): Attempt **ALL THREE (3)** questions.  
**Section B** (2 Questions): Attempt **ONE (1)** question **ONLY**.  
All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.
4. Show all steps.
5. Formulas are provided and attached in Appendix.

**SECTION A****Answer ALL THREE (3) Questions.****Question 1**

- a. Find the angle between the vectors  $u = i - 2j + 2k$  and  $v = -3i + 2j + k$ . [2 marks]
- b. Consider the points  $P = (-1, 2, 1)$ ,  $Q = (0, -3, 2)$ , and  $R = (1, 1, -4)$ .
- Find the vectors  $u = \vec{PQ}$  and  $v = \vec{PR}$ . [3 marks]
  - Compute the cross product of  $\vec{PQ}$  and  $\vec{PR}$ . [2 marks]
- c. Let  $K$  be the line passing through  $A(5,0,-2)$  that is parallel to the line joining  $B(2,3,-1)$  and  $C(-1,0,1)$ . Determine the parametric equation of the line  $K$ . [3 marks]

**Question 2**

- a. The first order differential equation (ODE) is given below:

$$3 \frac{dy}{dx} = \frac{x^3}{y^2 - 1}$$

- Show that the ODE is separable. [1 mark]
  - Solve the ODE using separable method. [2 marks]
- b. Find the general solution of the following homogeneous linear second order differential equation.
- $$2y'' - 4y' + 2y = 0$$
- [3 marks]
- c. Solve the exact differential equation,  $x^2 y^4 dx + \frac{4}{3} x^3 y^3 = 0$ . [4 marks]

**Question 3**

A researcher would like to investigate the relationship between the daily smart phone usage duration (measured in hours),  $x$ , and the sleep quality (measured in a certain Quality Sleep Index),  $y$ , among university students. The summary of the sample data collected by this researcher is as follows:

$$n = 12, \sum x = 124, \sum y = 124, \sum x^2 = 1578, \sum y^2 = 1568, \sum xy = 1045$$

- Compute  $\bar{x}, \bar{y}, S_{xy}$  and  $S_{xx}$ . [4 marks]
- Find the regression model,  $\hat{y} = \beta_0 + \beta_1 x$ , to fit the data. [3 marks]
- What is the value of correlation coefficient,  $r$ ? What does this value indicate? [3 marks]

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**SECTION B**

Answer ONE (1) Question ONLY.

**Question 4**

a. Given  $f(x) = \begin{cases} 6x-1, & x < 2 \\ 4x+3, & x \geq 2 \end{cases}$ .

Compute  $\lim_{x \rightarrow 2^-} f(x)$  and  $\lim_{x \rightarrow 2^+} f(x)$ . Hence, determine whether  $\lim_{x \rightarrow 2} f(x)$  exists by applying the suitable limit theorem(s) or rule(s). [3 marks]

b. By applying the appropriate technique(s) or rule(s), compute  $\lim_{x \rightarrow \infty} \frac{5x+1}{7x-3}$ . [2 marks]

c. By using the appropriate technique(s), find

i. the second derivative ( $y''$ ) of  $y = \frac{(x-1)(x^5-3x^2)}{x^2}$ . [2 marks]

[Hint: Simplify the function first may be easier to find the derivative.]

ii. the derivative of  $y = (2x^4 + e^x)^3$ . [Hint: Chain rule may be useful.] [1 mark]

iii.  $\frac{dy}{dx}$  for  $xy + \ln x = 0$ . [2 marks]

**Question 5**

a. Find  $\int \frac{(3x^2-4)^2}{x^2} dx$  by using appropriate integration technique. [2 marks]

b. Use the *Fundamental Theorem of Calculus* to compute  $\int_{-1}^0 (2x^3 - 2x + 3) dx$ . [2 marks]

c. Use *integration by substitution* to compute  $\int 3x^2 \sqrt{1+x^3} dx$ . [3 marks]

d. Use *integration by part* to compute  $\int 2x^3 \cos x dx$ . [3 marks]

Continued.....

## APPENDIX

1. Length of a vector  $\mathbf{v}$ :  $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$
2.  $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$
3. Dot product:  $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$
4. Parametric equation:  $x = x_0 + at$ ;  $y = y_0 + bt$ ;  $z = z_0 + ct$
5. Criterion for an Exact Differential Equation:  $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
6. General solution of the homogeneous linear second order differential equation
  - For Distinct real roots:  $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$
  - For Repeated roots:  $y = c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_1 x}$
  - For Complex conjugate roots:  $y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$ .
7. Derivative Formulas

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

**Power Rule:**  $\frac{d}{dx} x^n = nx^{n-1}$  if  $f(x) = x^n$  with  $n \in \mathbb{R}$

**Product Rule:**  $\frac{d}{dx} (f \cdot g) = f \frac{dg}{dx} + g \frac{df}{dx}$

**Chain Rule:** If  $y = f(u)$  and  $u = g(x)$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Continued.....

## 8. Integration Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ for } n \neq -1, n \text{ rational}$
$\int \frac{1}{x} dx = \ln x + C$
$\int e^x dx = e^x + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$
$\int \cos kx dx = \frac{\sin kx}{k} + C$
<b>Integration by Substitution:</b> If $u = g(x)$ and $du = g'(x)dx$ , then $\int f(u)du = \int f(g(x)) \frac{d}{dx} g(x) dx$
<b>Integration by Part:</b> $\int u dv = uv - \int v du$

$$9. \quad \bar{x} = \frac{\sum x}{n}$$

$$S_{xy} = \sum xy - \frac{(\sum x \sum y)}{n}, \quad S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}, \quad S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$10. \quad \beta_1 = \frac{S_{XY}}{S_{XX}}, \quad \beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$11. \text{ Sample correlation coefficient } = r = \frac{S_{xy}}{\sqrt{S_{xx}} \sqrt{S_{yy}}}$$

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